# Experimental Development of Quantitative Energy Expressions

Once students have developed a qualitative feel for the energy concept, they can develop the mathematical expressions for $E_{elastic}$, $E_{gravitational}$, $E_{kinetic}$, and $E_{transferred}$ by working through the following sequence of labs. One advantage of these labs is that they all use the same equipment, allowing student to focus on the physics rather than the variations in lab apparatus.

Apparatus: Cart and inclined track with a loop spring, pulley, masses, and photogate or motion detector.
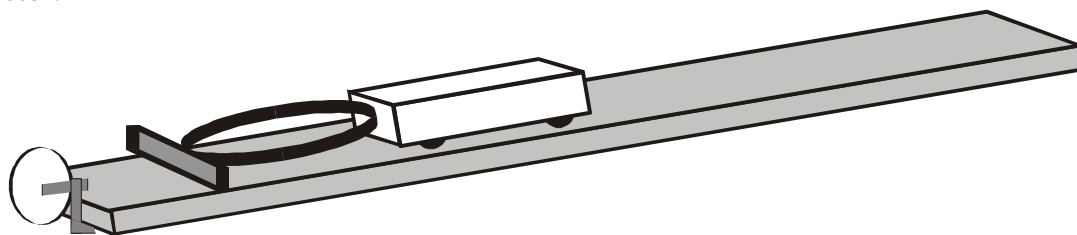
The loop spring is a piece of spring steel. It follows Hooke's law up to a compression of about half of its diameter. You can make the spring loops from the spring coil of a tape measure. Alternatively, Rex Rice (rex_rice@clayton.k12.mo.us) has offered to provide cut strips with a hole punched at each end for the cost of material and mailing. The threaded bolt passes through the holes in the end of the steel strip and is fastened with a nut. Two fender washers straddle the end stop and are tightened down with a wing nut.
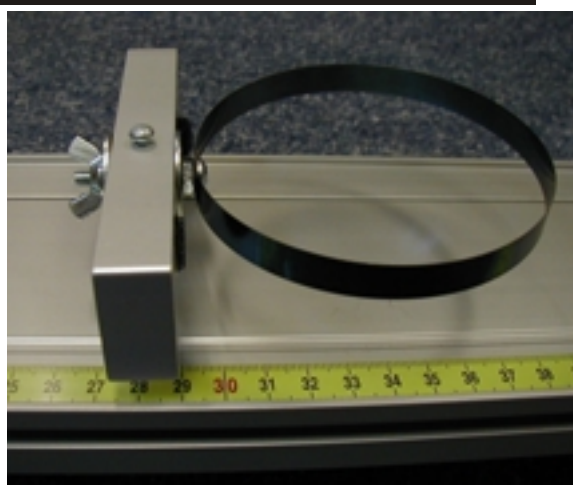
Lab 1. Students measure force vs. deformation (Hooke's Law)
Lab 2. Students measure shoot height vs. deformation
Lab 3. Students measure speed vs. drop height *or* E_{el} vs. speed.

Lab 1: Force vs. Deformation (Hooke's Law)

Pre-lab discussion
The Hooke's Law lab is used to introduce the quantification of elastic potential energy. Students know that as the force acting on an elastic system increases, so does the energy stored in the system. The purpose of this lab is to determine the relationship between the force and the amount of stretch of a spring.

Lab performance notes
- The tracks should be leveled so that the cart does not exert a force on the spring. A string tied to the cart passes over the pulley and ends in a loop. Students can hook masses on the end of the string and measure the change in position of the cart as the spring compresses.
- Students should be cautioned against over compressing the springs.
- When they graph their data, they should be advised to plot F vs x, despite the fact that force was the independent variable.

Post-lab discussion
Students should find that force is proportional to the deformation. The general equation for the graphs should be $F = kx + F_0$, where the slope, k, indicates the force per unit length of deformation, and the intercept, F_0, indicates how much force must be applied before the spring begins to stretch in a linear manner. *After* discussing the *meaning* of the slope and intercept, you can use the terms spring constant for k and loading force for F_0.

Lab 2: Shoot Height vs. Deformation

Pre-lab discussion
Now that they have discovered Hooke's Law, the students are ready for a discussion of the *energy* situation of the spring. A pie chart analysis of the situation as the spring is deformed shows the pies getting larger with increasing deformation, indicating more and more energy being stored with each increase in applied force. What could that stored energy do? It could shoot the cart horizontally, and the system energy would then be stored in $E_{kinetic}$. It could also shoot the cart up an inclined track so the system would eventually be stored in $E_{gravitational}$. Lead the students to investigate the latter first, as the analysis is a bit easier.

Lab performance notes
- Students are comparing spring deformation vs. the change in height of the cart. The change in height is most accurately determined by measuring the shoot distance along the track and using trigonometry to calculate the change in height.
- Make all measurements from the end of the cart not touching the spring to minimize procedural errors.
- Let the zero deformation, zero height position be where the cart rests when being supported by the spring.
- Students should use the same loop spring they used for their Hooke's law data.

Post-lab discussion
The goal is to use the experimental results to quantify the energy storage for the initial and final situations in this activity.
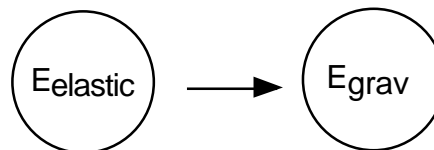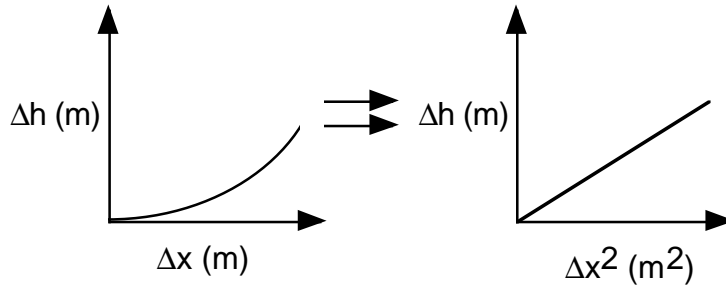
From the Hooke's Law experiment:
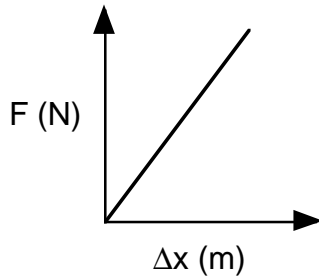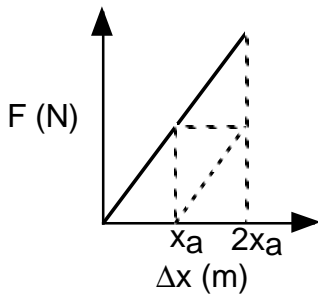
From the shoot height vs. deformation experiment:

$F = k\Delta x$

$\Delta h = A\Delta x^2$

where A is the slope of the linearized graph

We know that E_{el} is related to Δx and E_g is related to Δh.

Therefore $E_{el} = E_g$

$\Delta x^2 \propto \Delta h$ from this lab. We want to turn this into an equality.

Looking at the F vs. Δx graph, as x doubles, what property of the graph doubles? This quantity is proportional to E_{el}.

The area under the graph is proportional to E_{el}.

$E_{el} \propto 1/2\ F\Delta x$

$\propto 1/2\ (k\Delta x)\Delta x$

$\propto 1/2\ k\Delta x^2$

Returning to $E_{el} = E_g$

$1/2\ k\Delta x^2 \propto \Delta h$

Use dimensional analysis to get at the remaining terms

$(N/m)\ m^2 \propto m$ need units of Force - gravitational force is a plausible candidate

Therefore, we predict:

$1/2\ k\Delta x^2 \approx mg\Delta h$

Test the prediction by comparing to experiment:

$A = \Delta h/\Delta x^2$ from the lab. Our predicted relation suggests that $A = k/(2mg)$

Students should test this relation calculating A from their measured information and comparing it to their slope, A. (They should be equal within experimental uncertainty.)

Now we can conclude that:

$E_{elastic} = 1/2k\Delta x^2$ and $E_g = mgh$

Lab 3: Speed vs. Drop Height *or* E$_{el}$ vs. Speed

The next step is to develop an expression for kinetic energy. Two methods are presented here. The first requires a photogate and the second requires a motion detector.

## Option 1. Speed vs. Drop Height

Incline the track slightly and mount a photogate near the low end of the track. Add a flag to the cart to block the photogate beam as the cart rolls down the track. Starting the cart from higher positions will result in greater speeds.

Pre-lab discussion

• Define the system to be the cart, earth and track.
• Have students draw pie graphs to show the change in internal energy distribution as the cart rolls down the track.
• Ask what factors are likely to be involved in the kinetic energy of the cart. Students should suggest mass and velocity.
• Ask students how they will determine the height of the cart. They should use trigonometry as they did in the previous lab.

Post-lab discussion

Student graphs should look similar to the following:
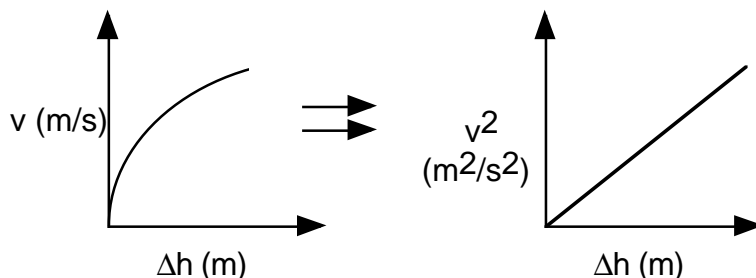
$v^2 = B \, \Delta h$
$v^2/ \, \Delta h = B$ students should note that B has units of m/s^2 and is very close to 2g.
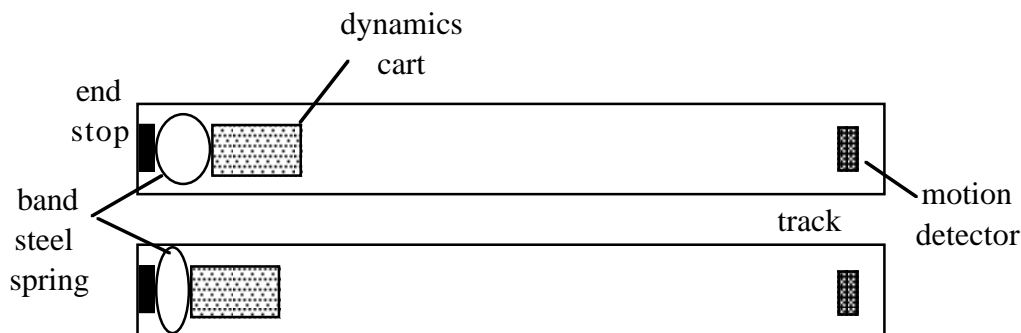$v^2 = 2g\Delta h$
$1/2 \, m \, v^2 = mg\Delta h$
therefore: $E_{kinetic} = 1/2 \, m \, v^2$

Option 2: E$_{el}$ vs. Speed

With a level track, push the cart into the spring with a known deformation. Release the cart and

measure its velocity with a photogate.

Lab performance notes
•   If students have already determined k for the spring earlier in the unit, they can move right into determining the relationship between energy and velocity.
•   Be sure to warn the students not to compress the spring past one half of its diameter.
•   Students should perform multiple trials and use the software (LoggerPro, Motion, Data Studio) help them determine the maximum velocity of the system for each trial. Students need to be careful to keep their hands out of the way of the motion detector.

Post-lab discussion
Students should obtain graphs similar to the ones below.
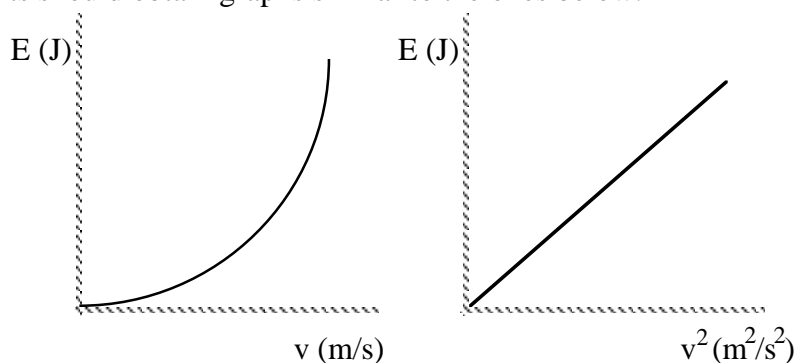
They might not immediately recognize that the units of slope reduce to kg. Once they realize this, remind them that the slope is usually related in some way to a variable held constant during the performance of the lab. Then, have them post their slopes and system masses. If they conducted their experiments carefully, they should recognize that the slope is roughly half of the system mass. This suggests that the expression for the kinetic energy of the system is $E_k = \frac{1}{2}mv^2$. Ask students why the slope is only approximately half of the mass; induce them to see that some energy is dissipated during the transfer. Next, ask if this is a random or directed error; they should be able to account for why their slopes are generally a bit small.

"Lab 4"
Students can now generalize the pattern they saw with the area under the F∆x graph by constructing a similar graph for lifting an object or dragging an object across the floor at constant velocity. In all cases, the area under the F∆x graph represents the energy transferred into the system by "working". Therefore, we can calculate energy transfer when we have a well defined force acting on essentially a point particle with the relation Work = F∆x.